Symmetry, Art, and Complex Analysis

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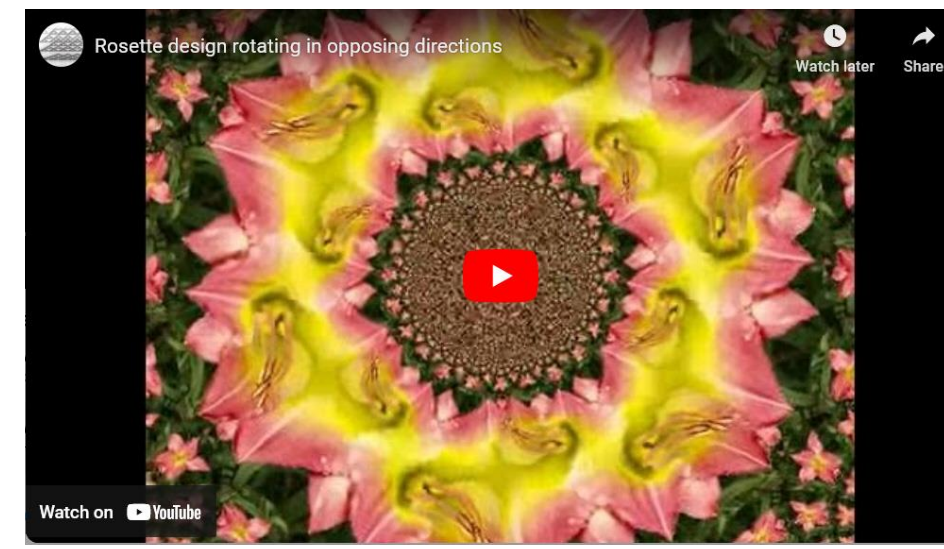
We use some fundamental ideas from complex analysis to create symmetric images and animations. Using a domain coloring algorithm, we generate mappings to the entire complex plane or the hyperbolic upper half-plane. The resulting designs can have rotational, translational, or mirror symmetry according to our chosen mapping functions. An appealing feature of these designs is how they reveal important properties of Euclidean and hyperbolic geometries. We can also generate animations of our designs. Our goal is to create designs and animations having significant artistic content.

The method we used is to start with a function $f(z)$ of a complex variable z, and then create a sum of terms of the form $f(S(z))$ using symmetry operations S characteristic of either Euclidean plane geometry or hyperbolic geometry. The design of such a symmetrized function g is created by using colors from points within a photo that are image points in the graph of g.

Egyptian Lotus Rosette

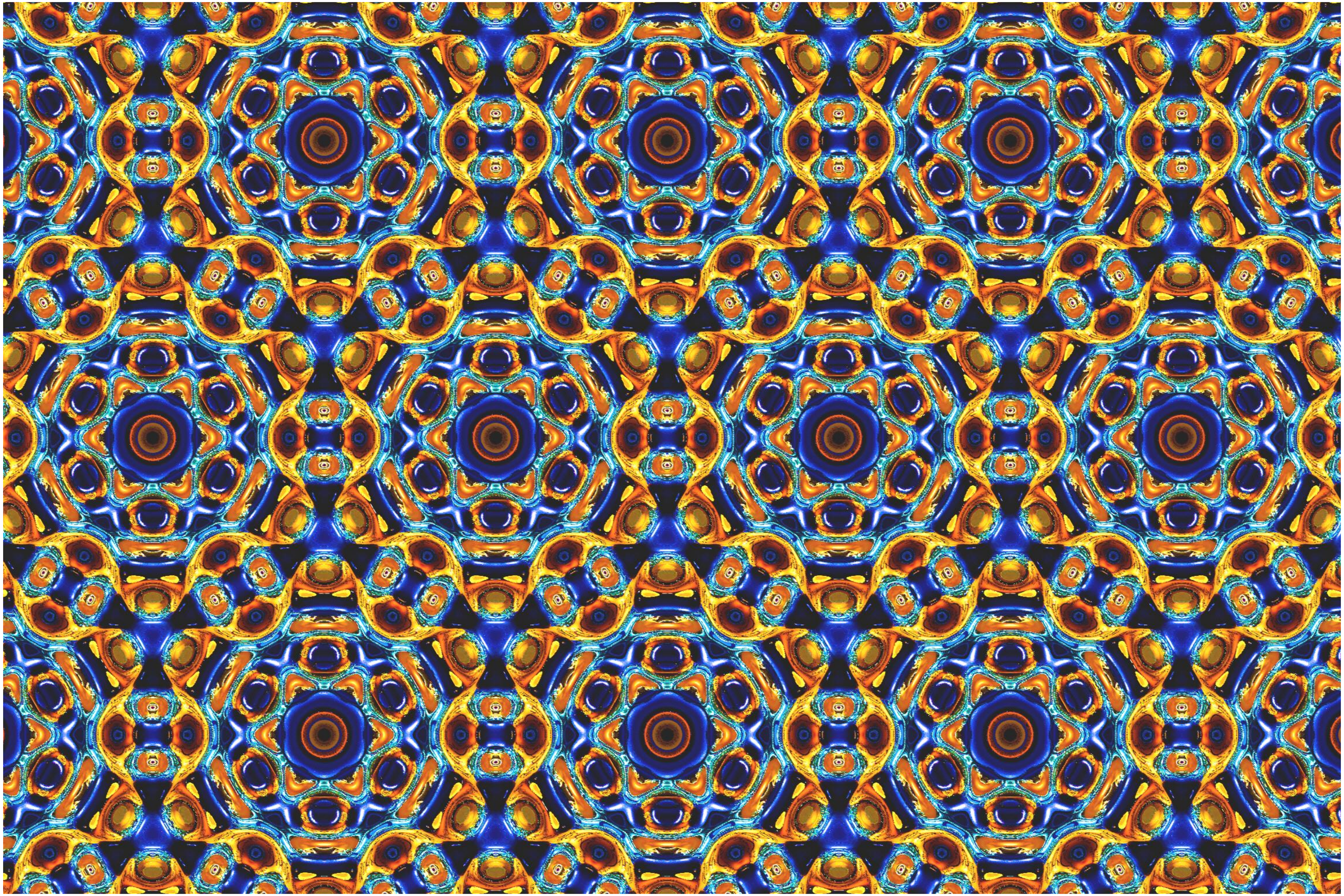
The image on the left is a square framing of a 6-fold rotationally symmetric design. The square framing creates an ambiguity between the 4-fold rotational symmetry of the square frame vs. the 6-fold rotational symmetry of the design.

The screenshot below is of a YouTube animation of a similar design. You can watch this animation by using the QR-code or URL.



https://www.youtube.com/watch?v=Pf1vJPywXWs

Crystalline World

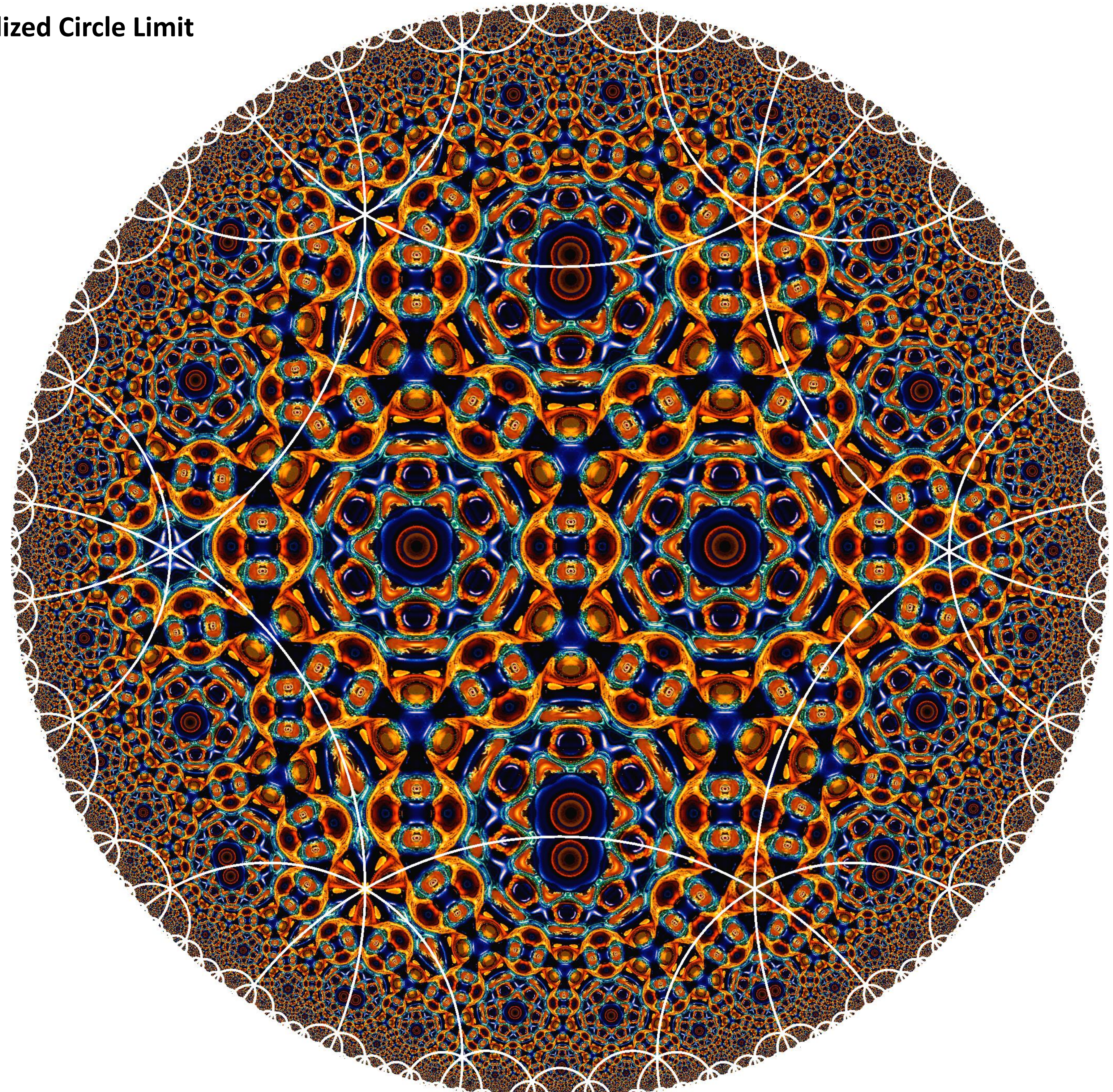


A design with 6-fold rotational symmetry, horizontal and vertical translation symmetry, and 6-fold mirror symmetries. You may observe a fluctuation between circular regions having 3-fold rotational symmetry vs. intersecting curved edge hexagonal regions having 6-fold rotational symmetry. This ambiguity in our visual perception of the design gives it some artistic weight.

Hyperbolic Sunset

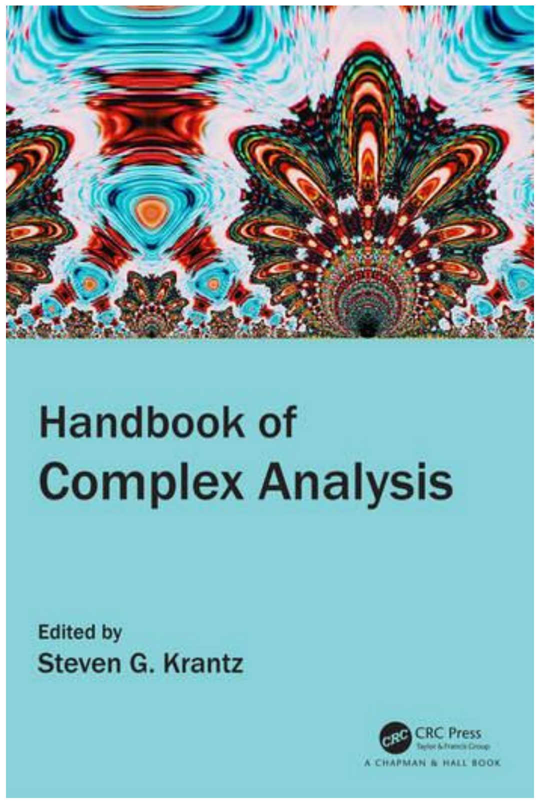
This design exhibits symmetry in hyperbolic geometry. The thin black circular arcs in the middle of the design are portions of geodesics, which play the role of lines in non-Euclidean hyperbolic geometry. In our paper we describe how these geodesics mark off the boundaries of a tessellation above the bottom border line, which is a line at infinity in hyperbolic geometry.

Crystallized Circle Limit



This design is a tessellation of the inside of a circle in hyperbolic geometry. The most famous examples of this kind of tessellation are the wood block prints created by M.C. Escher. The QR-code below links to a Wikipedia article that shows the Escher print *Circle Limit III*.

https://en.wikipedia.org/wiki/Circle_Limit_III



A complete description of our work is given in our paper, *Symmetry and Art*, in the **Handbook on Complex Analysis,** CRC Press, 2022. The cover of the book is shown on the left. One of our designs was used for its cover art. A preprint of our paper, as well as more information, can be obtained from the QR-code, or URL, shown on the right.

https://www.jameswalkermathmusic.net/art/